Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Monday 23 October 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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P 7 5 7 1 0 A 0 1 2 8

Pearson

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\frac{8}{(2-5x)^2}$$

writing each term in simplest form.

(4)

(b) Find the range of values of x for which this expansion is valid.

(1)

(a) Get the formula for the Binomial Expansion from the formula booklet:

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$, Range of validity $|x|<1$

We see that the given expansion is in form $(a+bx)^n$, we need to manipulate to get $(1+ax)^n$.

$$8(2-5x)^{-2} = 8(2)^{-2}(1-\tfrac{5}{2}x)^{-2}$$
$$= 2(1-\tfrac{5}{2}x)^{-2}$$

Substitute into the Formula: B1

$$2(1-\tfrac{5}{2}x)^{-2} = 2\left[1 + (-2)(-\tfrac{5}{2}x) + \frac{(-2)(-3)}{2!}(-\tfrac{5}{2}x)^2 + \frac{(-2)(-3)(-4)}{3!}(-\tfrac{5}{2}x)^3 + \dots\right]$$ M1A1

$$= 2\left(1 + 5x + 3\left(\tfrac{25}{4}x^2\right) + (-4)\left(-\tfrac{125}{8}x^3\right) + \dots\right)$$

$$= 2\left(1 + 5x + \tfrac{75}{4}x^2 + \tfrac{125}{2}x^3\right)$$

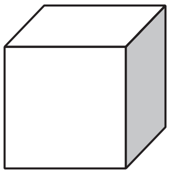
$$= 2 + 10x + \tfrac{75}{2}x^2 + 125x^3 + \dots$$ A1

(b) Range of validity for $(1-x)^n$ is $-1<x<1$ $\therefore$ for $(2-5x)^{-2}$ it's: $-\frac{2}{5} < x < \frac{2}{5}$ B1

Question 1 continued

(Total for Question 1 is 5 marks)

2.



x cm

Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the surface area of the cube is $S\,\text{cm}^2$
- the volume of the cube is $V\,\text{cm}^3$

Given that the surface area of the cube is increasing at a constant rate of $4\,\text{cm}^2\,\text{s}^{-1}$

(a) show that $\dfrac{dx}{dt} = \dfrac{k}{x}$ where k is a constant to be found,

(4)

(b) show that $\dfrac{dV}{dt} = V^p$ where p is a constant to be found.

(3)

(a) We're given that $\dfrac{dS}{dt} = 4\text{cm}^2\text{s}^{-1}$ B1

Also, surface area of a cube, $S = 6x^2$. B1

Differentiate this: $\dfrac{dS}{dx} = 12x$.

We want $\dfrac{dx}{dt}$: $\dfrac{dx}{dt} = \dfrac{dx}{dS} \times \dfrac{dS}{dt}$ M1

$\dfrac{dx}{dt} = \dfrac{1}{12x} \times 4 = \dfrac{1}{3x}$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{1}{3x}$ A1

(b) Volume of a cube $\rightarrow V = x^3$, $\dfrac{dV}{dx} = 3x^2$. B1

found in (a)

To get $\dfrac{dV}{dt}$: $\dfrac{dV}{dt} = \dfrac{dV}{dx} \times \dfrac{dx}{dt}$ M1

$= \cancel{3}x^{\cancel{2}\,1} \times \dfrac{1}{\cancel{3x}} = x$ $\qquad V = x^3 \rightarrow x = V^{\frac{1}{3}}$.

$\therefore \dfrac{dV}{dt} = V^{\frac{1}{3}}$ A1

Question 2 continued

(Total for Question 2 is 7 marks)

Turn over

3. **In this question you must show all stages of your working.**

Solutions based on calculator technology are not acceptable.

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x}\, dx$$

giving your answer in simplest form.

(5)

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2}\, dx = a + \ln b$$

where a and b are constants to be found.

(7)

(i) $\int x^2 e^{2x} dx$ Multiplication, ∴ use by Parts

★ Integration by Parts:

$\int vu'dx = vu - \int uv'dx$

OR

SDI table method

Method I : Formula

$\int x^2 e^{2x} dx$

$v = x^2 \xrightarrow{\frac{d}{dx}} v' = 2x$

$u = \frac{e^{2x}}{2} \xleftarrow{\int} u' = e^{2x}$

$= \frac{1}{2}x^2 e^{2x} - \int xe^{2x} dx$ multiplication ∴ By Parts again

M1

$u = x \xrightarrow{\frac{d}{dx}} u' = 1$

$v = \frac{e^{2x}}{2} \xleftarrow{\int} v' = e^{2x}$

$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} - \int \frac{e^{2x}}{2} dx$

M1

$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{e^{2x}}{4}$ A1

add limits $\int_0^4 x^2 e^{2x} dx = \left[\frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{e^{2x}}{4}\right]_0^4$ M1

$= 8e^8 - 2e^8 + \frac{e^8}{4} - \frac{1}{4} = \frac{25e^8}{4} - \frac{1}{4}$ A1

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Question 3 continued

Method II: SDI

$\int x^2 e^{2x}\,dx$:

SDI – table

Sign	Differentiate	Integrate
+	x^2	e^{2x}
−	$2x$	$\frac{1}{2}e^{2x}$
+	2	$\frac{1}{4}e^{2x}$
−	0	$\frac{1}{8}e^{2x}$

$\Rightarrow \int x^2 e^{2x}\,dx = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{e^{2x}}{4} + c$

M1 M1 A1

$$\int_0^4 x^2e^{2x}\,dx = \left[\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{e^{2x}}{4}\right]_0^4 \quad \text{M1}$$

$$= 8e^8 - 2e^8 + \frac{e^8}{4} - \frac{1}{4}$$

$$= \frac{25e^8}{4} - \frac{1}{4} \quad \text{A1}$$

ii. Use substitution $u = 2x-1$, we need to find new limits, $dx = \ldots$ and $x = \ldots$

New Limits:

x	u
$\frac{21}{2} - 2(\frac{21}{2}) - 1 \rightarrow$	20
$3 - 2(3) - 1 \rightarrow$	5

To get dx, differentiate $u = 2x-1$

$\frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$ B1

Get $x = \ldots$:

$x = \frac{u+1}{2}$

Substitute into the Integration:

$$\int_5^{20} \frac{4\left(\frac{u+1}{2}\right)}{u^2} \cdot \frac{du}{2}$$

$$= \int_5^{20} \frac{\cancel{2}u + \cancel{2}}{\cancel{2}u^2}\,du \quad \text{M1A1}$$

$$= \int_5^{20} \frac{u+1}{u^2}\,du$$

$$= \int_5^{20} \frac{1}{u} + \frac{1}{u^2}\,du$$

split the fraction

$$= \left[\ln u - \frac{1}{u}\right]_5^{20} \quad \text{dM1A1}$$

$\int \frac{1}{x}\,dx = \ln x + c$

$$= \ln 20 - \frac{1}{20} - \ln 5 + \frac{1}{5} \quad \text{M1}$$

apply ln law: $\ln a - \ln b = \ln\frac{a}{b}$

$$= \ln 4 + \frac{3}{20} \quad \text{A1}$$

Question 3 continued

Question 3 continued

(Total for Question 3 is 12 marks)

4. (a) Prove by contradiction that for all positive numbers k

$$k + \frac{9}{k} \geqslant 6$$

(4)

(b) Show that the result in part (a) is not true for all real numbers.

(1)

(a) Make an Assumption: We want our assumption to be the opposite of what we are trying to prove.

"Assume that there is a positive number k that satisfies $k+\frac{9}{k} < 6$" B1

In the Proof, we are trying to contradict the assumption

Try to Solve: $k + \frac{9}{k} < 6$

$k^2 + 9 < 6k$

$k^2 - 6k + 9 < 0$ M1

$(k-3)^2 < 0$ A1

However all squared numbers are ≥ 0, $\therefore (k-3)^2 < 0$ has no real solutions, $(k-3)^2 \geq 0$.

Hence, $k + \frac{9}{k} \geq 6$, proven by contradiction. A1

(b) $k + \frac{9}{k} \geq 6$

$k = -1 \Rightarrow -1 + \frac{9}{-1} = -10$

$-10 < 6$ $\therefore$ not true for all real numbers

B1

Question 4 continued

(Total for Question 4 is 5 marks)

5.

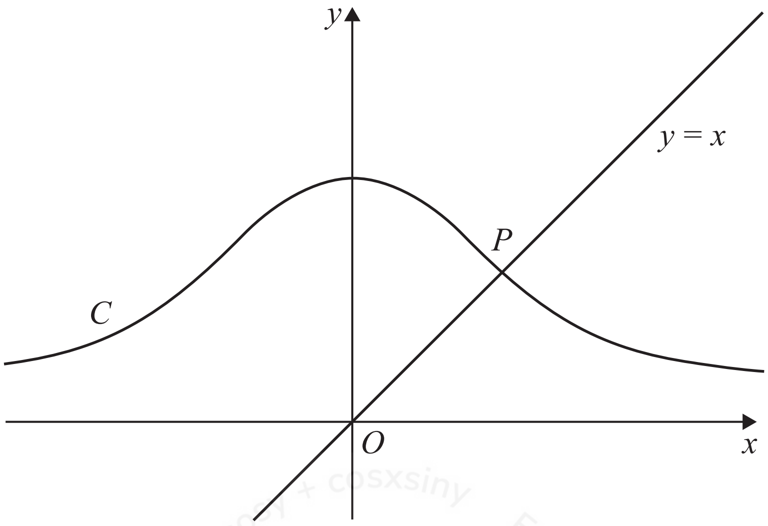


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y^3 - x^2 + 4x^2y = k$$

where k is a positive constant greater than 1

(a) Find $\frac{dy}{dx}$ in terms of x and y. **(5)**

The point P lies on C.

Given that the normal to C at P has equation $y = x$, as shown in Figure 2,

(b) find the value of k. **(5)**

(a) ★ Implicit differentiation:

When differentiating y, multiply by $\frac{dy}{dx}$

$y^3 - x^2 + 4x^2y = k$

Multiplication ∴ product rule:

$\frac{d}{dx}(uv) = uv' + u'v$

$u = 4x^2 \quad u' = 8x$

$v = y \quad v' = \frac{dy}{dx}$

$3y^2\frac{dy}{dx} - 2x + 4x^2\frac{dy}{dx} + 8xy = 0$ M1M1

$\Rightarrow 3y^2\frac{dy}{dx} + 4x^2\frac{dy}{dx} = 2x - 8xy$ A1

$\frac{dy}{dx}(3y^2 + 4x^2) = 2x - 8xy$ M1

$\frac{dy}{dx} = \frac{2x - 8xy}{3y^2 + 4x^2}$ A1

Question 5 continued

(b) Since the normal has equation $y=x$ (given) the tangent must be $y=-x$, hence at P, $\frac{dy}{dx}=-1$.

$$\frac{dy}{dx}=\frac{2x-8xy}{3y^2+4x^2}=-1 \quad \text{M1}$$

Also, since at P $y=x$:

$$\frac{2x-8x^2}{3x^2+4x^2}=-1 \qquad \text{(Replace all } y \text{ with } x\text{)}$$

M1

$$2x-8x^2=-3x^2-4x^2$$

$$2x=x^2$$

$$2=x=y \quad \text{A1}$$

Use this and the given equation for C to get k:

$$2^3-2^2+4(2)^3=k \quad \text{dd M1}$$

$$k=36 \quad \text{A1}$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

direction vectors

6. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point P

(a) state the coordinates of P

(1)

Given that the angle between lines l_1 and l_2 is θ

(b) find the value of $\cos\theta$, giving the answer as a fully simplified fraction.

(3)

The point Q lies on l_1 where $\lambda = 6$

Given that point R lies on l_2 such that triangle QPR is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle QPR

(3)

(d) find the coordinates of the possible positions of point R

(3)

(a) By observing the equations:

$P(2, 3, -7)$

B1

(b) Use Formula:

$\cos\theta = \left| \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|} \right|$ Formula for dot product: $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$

use the direction vectors:

$|\bar{a}| = \sqrt{1^2 + 2^2 + 2^2} = 3$

$|\bar{b}| = \sqrt{4^2 + (-1)^2 + 8^2} = 9$

$\bar{a} \cdot \bar{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} = 4 - 2 + 16 = 18$

M1

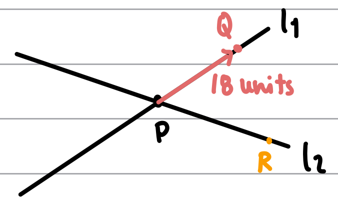
$\rightarrow \cos\theta = \dfrac{18}{3 \cdot 9} = \dfrac{2}{3}$ dM1

$\cos\theta = \dfrac{2}{3}$ A1

P 7 5 7 1 0 A 0 1 6 2 8

Question 6 continued

(c) Diagram



Point Q: $\vec{OQ} = \begin{pmatrix}2\\3\\-7\end{pmatrix} + 6\begin{pmatrix}1\\2\\2\end{pmatrix} = \begin{pmatrix}8\\15\\5\end{pmatrix}$

$\vec{PQ} = \vec{OQ} - \vec{OP}$

$= \begin{pmatrix}8\\15\\5\end{pmatrix} - \begin{pmatrix}2\\3\\-7\end{pmatrix} = \begin{pmatrix}6\\12\\12\end{pmatrix}$

$|\vec{PQ}| = \sqrt{6^2+12^2+12^2} = 18$ units M1

Since the triangle is isosceles, $|\vec{PR}|$ must also be 18 units

$\therefore$ Use formula: area $= \frac{1}{2}ab\sin C$.

$PQR = \frac{1}{2}(18)(18)\left(\sqrt{1-\left(\frac{2}{3}\right)^2}\right)$ Use $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$

M1

$= 54\sqrt{5}$ units^2 A1

(d) $\vec{PR} = \begin{pmatrix}4\mu\\\mu\\8\mu\end{pmatrix}$

Since $|\vec{PR}| = |\vec{PQ}|$:

$(4\mu)^2 + \mu^2 + (8\mu)^2 = 6^2 + 12^2 + 12^2$ M1

$16\mu^2 + \mu^2 + 64\mu^2 = 324$

$81\mu^2 = 324$

$\mu^2 = 2$, $\therefore \mu = \pm 2$

Substitute $\mu = \pm 2$ back into equation of l_2:

$r = \begin{pmatrix}2\\3\\-7\end{pmatrix} + \mu\begin{pmatrix}4\\-1\\8\end{pmatrix}$

$\mu = 2$

$\vec{OR} = \begin{pmatrix}2\\3\\-7\end{pmatrix} + \begin{pmatrix}8\\-2\\16\end{pmatrix}$ dM1

$= \begin{pmatrix}10\\1\\9\end{pmatrix}$

$(10, 1, 9)$

$\mu = -1$

$\vec{OR} = \begin{pmatrix}2\\3\\-7\end{pmatrix} + \begin{pmatrix}-8\\2\\-16\end{pmatrix}$

$= \begin{pmatrix}-6\\5\\-23\end{pmatrix}$

$(-6, 5, -23)$

Possible coordinates: $(10, 1, 9)$ and $(-6, 5, -23)$ A1

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x, in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

(a) Show that $k = 1.8$

(2)

(b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x, in thousands, is modelled by the differential equation

$$3\frac{dx}{dt} = x(9 - 2x)$$

(c) Write $\dfrac{3}{x(9 - 2x)}$ in partial fraction form.

(3)

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

(5)

(e) Find the long-term population of goats predicted by this **second** model.

(1)

(a) at $t=0, x=3$:

$$3 = \frac{k(9(0)+5)}{4(0)+3} \quad \text{M1}$$

$$3 = \frac{5k}{3} \Rightarrow \frac{9}{5} = k, \quad \boxed{k = 1.8 \text{ shown}}$$

A1

(b)

$$x = \frac{1.8(9t+5)}{4t+3}$$

$$= \frac{1.8\left(9 + \frac{5}{t}\right)}{4 + \frac{3}{t}} \quad \text{divide everything by } t.$$

as $t \longrightarrow \infty$: $\frac{5}{t} \to 0$, $\frac{3}{t} \to 0$. $\therefore$ $x = \dfrac{1.8(9+0)}{4+0} = 4.05$ thousand.

$\boxed{4050 \text{ goats.}}$ B1

Question 7 continued

(c) We are asked to do Partial Fractions

$$\frac{3}{x(9-2x)} \equiv \underbrace{\frac{A}{x} + \frac{B}{9-2x}}_{}$$ B1

Manipulate this to get common denominator

$$\frac{3}{x(9-2x)} = \frac{A(9-2x)+Bx}{x(9-2x)}$$

Equate the numerators and substitute in values in order to get A and B

$$3 = A(9-2x) + Bx$$

choose values to substitute in so that brackets become 0, to make your life easier!

$x=0 \Rightarrow 3 = A(9) + B\cancel{(0)}^{0}$

$A = \frac{1}{3}$

$x = \frac{9}{2} \Rightarrow 3 = A\cancel{(0)}^{0} + \frac{9}{2}B$ M1

$B = \frac{2}{3}$

$\therefore \frac{3}{x(9-2x)} = \frac{1}{3x} + \frac{2}{3(9-2x)}$ A1

Question 7 continued

(d) $$3\frac{dx}{dt} = x(9-2x)$$

$$\int \frac{3}{x(9-2x)}\,dx = \int 1\,dt$$ Separate Variables by grouping the like terms on one side

$$\int \frac{1}{3x} + \frac{2}{3(9-2x)}\,dx = \int 1\,dt$$ use the P.F. we did in (c)

$$\frac{1}{3}\int \frac{1}{x} + \frac{2}{9-2x}\,dx = \int 1\,dt$$

$$\frac{1}{3}\left(\ln x + \frac{2}{-2}\ln(9-2x)\right) = t + c$$ Integrate

Reverse chain rule: divide by derivative of the bracket

$$\frac{1}{3}\ln x - \frac{1}{3}\ln(9-2x) = t + c$$ M1A1

Substitute $t=0,\ x=3$

$$\cancel{\frac{1}{3}\ln 3} - \cancel{\frac{1}{3}\ln 3} = 0 + c$$

$$c = 0$$ M1

$$\therefore \frac{1}{3}\ln x - \frac{1}{3}\ln(9-2x) = t$$ Rearrange to make x subject

$$\frac{1}{3}\ln\left(\frac{x}{9-2x}\right) = t$$ $\ln a - \ln b = \ln\frac{a}{b}$

$$\ln\left(\frac{x}{9-2x}\right) = 3t$$

$$e^{\ln\left(\frac{x}{9-2x}\right)} = e^{3t}$$

ddM1 $$\frac{x}{9-2x} = e^{3t}$$ $e^{\ln x} = x$

$$\frac{9-2x}{x} = e^{-3t}$$

$$\frac{9}{x} - 2 = e^{-3t}$$

$$\frac{9}{x} = 2 + e^{-3t}$$

$$x = \frac{9}{2+e^{-3t}}$$ A1

(e) As $t \to \infty$: $$x = \frac{9}{2}$$

$\therefore$ 4500 goats B1

Question 7 continued

(Total for Question 7 is 12 marks)

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Turn over

8.

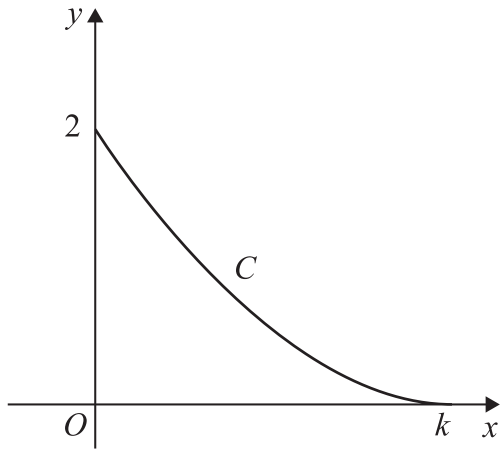

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the y-axis at 2 and the x-axis at k, where k is a constant.

(a) State the value of k.

(1)

(b) Use parametric differentiation to show that

$$\frac{dy}{dx} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The tangent to C at the point P cuts the y-axis at the point N.

(c) Find the exact y coordinate of N, giving your answer in simplest form.

(3)

The region bounded by the curve, the x-axis and the y-axis is rotated through 2π radians about the x-axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta(1 - \cos 4t)\, dt$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

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P 7 5 7 1 0 A 0 2 4 2 8

Question 8 continued

(a) at x-axis, $y=0$.

$$0 = 2\cos t$$

$t = \frac{\pi}{2}$ t value when x-axis is met

$$x = 6\left(\frac{\pi}{2}\right) - 3\cancel{\sin(\pi)}^{0}$$

$x = 3\pi$ B1

(b) ★ Parametric differentiation:

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ get these by differentiating the parametrics separately.

$y = 2\cos t$

$\frac{dy}{dt} = -2\sin t$

$x = 6t - 3\sin 2t$

$\frac{dx}{dt} = 6 - 6\cos 2t$

$\frac{dy}{dx} = \frac{-2\sin t}{6 - 6\cos 2t}$ M1A1

$= \frac{-2\sin t}{6 - 6(1 - 2\sin^2 t)}$ use double angle formula: $\cos 2t = 1 - \sin^2 t$

$= \frac{-2\sin t}{6 - 6 + 12\sin^2 t}$

$= \frac{-\cancel{2}\cancel{\sin t}}{6\cancel{12}\sin^{\cancel{2}} t}$

$= -\frac{1}{6\sin t}$ $\frac{1}{\sin t} = \operatorname{cosec} t$

$= -\frac{1}{6}\operatorname{cosec} t$ M1A1

P 7 5 7 1 0 A 0 2 5 2 8

Question 8 continued

(c) Get P in Cartesian:

$y = 2\cos\frac{\pi}{4} = \sqrt{2}$

$x = 6\left(\frac{\pi}{4}\right) - 3\sin\frac{\pi}{2} = \frac{3\pi}{2} - 3$

$P\left(\frac{3\pi}{2} - 3, \sqrt{2}\right)$ B1

Get the gradient at $\frac{\pi}{4} = t$:

$$\frac{dy}{dx} = -\frac{1}{6}\operatorname{cosec}\frac{\pi}{4} = -\frac{\sqrt{2}}{6}$$

Use $y - y_1 = m(x - x_1)$ to get the equation of the line

$$y - \sqrt{2} = -\frac{\sqrt{2}}{6}\left(x - \left(\frac{3\pi}{2} - 3\right)\right)$$ M1

We want y at $x=0$:

$$y - \sqrt{2} = -\frac{\sqrt{2}}{6}\left(0 - \left(\frac{3\pi}{2} - 3\right)\right)$$

$$y = \frac{\sqrt{2}}{6}\left(\frac{3\pi}{2} - 3\right) + \sqrt{2}$$

$$= \frac{\pi\sqrt{2}}{4} - \frac{\sqrt{2}}{2} + \sqrt{2}$$

$$y_N = \frac{\pi\sqrt{2}}{4} + \frac{\sqrt{2}}{2}$$ A1

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Question 8 continued

(d)i. Formula for Volume of Revolution around the x-axis:

$$V = \pi\int_a^b y^2\,dx$$

AND

★ Parametric Integration

$$\text{area} = \int_a^b y\frac{dx}{dt}\,dt$$

where a and b are the points on the x-axis bounding the area

$$\therefore V = \pi\int_0^{\frac{\pi}{2}} y^2\frac{dx}{dt}\,dt$$

$$= \pi\int_0^{\frac{\pi}{2}} (2\cos t)^2(6-6\cos 2t)\,dt$$

$$= \pi\int_0^{\frac{\pi}{2}} 4\cos^2 t\cdot 6(1-\cos 2t)\,dt \quad \text{M1A1}$$

$$= \pi\int_0^{\frac{\pi}{2}} 2(\cos 2t+1)\cdot 6(1-\cos 2t)\,dt$$

double angle formula: $\cos 2A = 2\cos^2 A - 1$

$2\cos^2 A = \cos 2A + 1$

$$= \pi\int_0^{\frac{\pi}{2}} 12\cdot(1+\cos 2t)(1-\cos 2t)\,dt$$

$$= \pi\int_0^{\frac{\pi}{2}} 12(1-\cos^2 2t)\,dt$$

$$= \pi\int_0^{\frac{\pi}{2}} 6(2-2\cos^2 2t)\,dt$$

double angle formula: $2\cos^2 A = \cos 2A + 1$

$$= \pi\int_0^{\frac{\pi}{2}} 6(2-\cos 4t-1)\,dt \quad \text{dM1}$$

$$= \pi\int_0^{\frac{\pi}{2}} 6(1-\cos 4t)\,dt$$

$$= \int_0^{\frac{\pi}{2}} 6\pi(1-\cos 4t)\,dt \quad \text{A1}$$

ii.

$$V = 6\pi\int_0^{\frac{\pi}{2}} 1-\cos 4t\,dt$$

$$= 6\pi\left[t-\frac{1}{4}\sin 4t\right]_0^{\frac{\pi}{2}} \quad \text{M1}$$

$$= 6\pi\left(\left(\frac{\pi}{2}-\frac{1}{4}\cancelto{0}{\sin 2\pi}\right)-\left(0-\frac{1}{4}\cancelto{0}{\sin(0)}\right)\right)$$

$$= 6\pi\left(\frac{\pi}{2}\right)$$

$= 3\pi^2$ volume

A1

Question 8 continued

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS